

Improved limits on $n \rightarrow n'$ transformation from the Spallation Neutron Source

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Conversions between neutrons n and dark matter candidate sterile neutrons n' have been proposed as a mechanism for baryon number $\mathcal{B}$ violation. In the case that there is a small mass difference Δm between the n and the n' states, oscillations can be induced by compensating for Δm with a magnetic field. A search for such neutron oscillations was performed at the Spallation Neutron Source by looking for anomalous neutron transmission through a strongly absorbing cadmium wafer inside of a 6.6 T magnet. The approach described here saw no regenerated neutrons above background, which provides an improved limit for neutron–sterile neutron transformations for a range of Δm between 0.1 and 1000 neV.

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I. INTRODUCTION

A. Motivation

Much of the matter in the Universe is invisible, not interacting via the electromagnetic force. This “dark matter” has thus far evaded direct detection, which has led to a robust suite of experimental and theoretical proposals looking for new exotic particles and interactions [1,2]. The traditional cold dark matter paradigm, featuring nonrelativistic matter interacting only by gravity, stands in tension with the observations of the density profiles of the dark matter halos of galaxies. One possible solution to this cusp/core problem could be through new forces acting between dark matter particles with astrophysical limits that vary depending on the complexity of interactions [3–5].

While the dark matter hypothesis can account for observations at the Galactic scale, physical mysteries also appear at much smaller distances. The weak interaction in the Standard Model (SM) of particle physics is left handed and maximally breaks parity. Early attempts to resolve this apparent asymmetry in chirality posited a right-handed

equivalent of the weak interaction and subsequently right-handed copies of the SM [6,7]. This right-handed “Mirror Model” (SM') would completely duplicate the $SU(3) \times SU(2) \times U(1)$ form of the SM with a dark sector denoted as $SU'(3) \times SU'(2) \times U'(1)$ [8].

Mirror matter, having the same interactions as the SM, could have similar complexity to observable matter on cosmological scales [9–11]. Any existing mirror matter would interact with normal matter predominately through gravity. The added particles and forces would have implications for element formation in the early Universe [12] and could even resolve discrepancies in the observed Hubble constant [13]. This could potentially lead to the formation of mirror-matter-based “dark stars,” or mixed stars with a dark matter subcomponent [14,15]. Such exotic objects could be sought with existing astrophysical dark matter searches [16,17].

Another major unsolved question in physics is the Universe’s matter-antimatter asymmetry [18], which requires a violation of baryon number $\mathcal{B}$ [19]. In a universe with mirror matter, SM neutral particles such as photons, the neutron n , or neutrinos ν , could mix with their mirror counterparts leading to observable oscillations. Such mixing would cause $\mathcal{B}$ violation in the neutron sector or $\mathcal{L}$ violation in the neutrino sector [20,21]. The neutron, due to its relatively long lifetime and relative ease of trapping, is a natural candidate for studying mirror matter in the laboratory. Neutrons would oscillate into their mirror

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counterpart n' , with some characteristic mixing mass denoted as $\epsilon_{nn'}$ [22]. Outside of the mirror model paradigm, the search for $n \rightarrow n'$ transformation also has been proposed as a probe of multiple branes [23]. This mixing, as a $\Delta B = 1$ process, can be contrasted with the well-studied neutron-antineutron $n \rightarrow \bar{n}$ oscillations, where $\Delta B = 2$ for $n \rightarrow \bar{n}$ [24,25]. Mirror neutrons could also be an intermediary process to $n \rightarrow \bar{n}$ conversion, $n \rightarrow n' \rightarrow \bar{n}$, which remains a relatively unexplored parameter space [26].

B. Phenomenology of n to n' transformation

In the absence of mixing with mirror neutrons, the evolution of the nonrelativistic neutron as it propagates in matter in the presence of an external magnetic field $\vec{B}$ and undergoes β decay can be described by the non-Hermitian Hamiltonian (in natural units $c = \hbar = 1$) (see, e.g., [27])

$$\mathcal{H}_n = m_n + \frac{p_n^2}{2m_n} + \mu_n \vec{\sigma} \cdot \vec{B} + V - i \left(W + \frac{\Gamma}{2} \right), \quad (1)$$

where m_n and μ_n are mass and magnetic moment of the neutron, $\vec{\sigma}$ is the vector of Pauli matrices, V is the real and W the imaginary absorptive parts of the optical potential, and Γ is the width of neutron decay. Equation (1) can be extended to the mirror sector, where the mirror neutron mass $m_{n'}$, decay width Γ' , and magnetic dipole moment $\mu_{n'}$ are not necessarily the same as for the ordinary neutron. There could exist a mirror magnetic field B' and a mirror-matter-induced spin-independent optical potential $V' - iW'$ [28],

$$\mathcal{H}_{n'} = m_{n'} + \frac{p_{n'}^2}{2m_{n'}} + \mu_{n'} \vec{\sigma} \cdot \vec{B}' + V' - i \left(W' + \frac{\Gamma'}{2} \right). \quad (2)$$

The states of the neutron from Eq. (1) and the mirror neutron from Eq. (2) would exist independently if there is no mixing interaction leading to oscillation between these states.

Evolution of the two-state (n, n') system follows the Schrödinger equation $i \frac{d\Psi}{dt} = \mathcal{H}\Psi$, where Ψ is the wave function of the n and n' components in two $(\pm)$ spin polarization states $\Psi = (\psi_n^+, \psi_n^-, \psi_{n'}^+, \psi_{n'}^-)^T$. The transformational evolution of the (n, n') system can then be described by introducing an unknown mixing parameter $\epsilon_{nn'}$ to the 4×4 Hamiltonian in the Schrödinger equation,

$$i \frac{d}{dt} \Psi = \begin{pmatrix} \mathcal{H}_n & \epsilon_{nn'} \\ \epsilon_{nn'} & \mathcal{H}_{n'} \end{pmatrix} \Psi. \quad (3)$$

The mass mixing between the two states, $\epsilon_{nn'}$, can be alternately described using the characteristic oscillation time $\tau_{nn'} = \epsilon_{nn'}^{-1}$. In a more general case, the mixing element $\epsilon_{nn'}$ can contain a transition magnetic moment $\eta_B \vec{\sigma}$ or electric dipole moment $\eta_E \vec{\sigma}$, which could couple to the

respective standard and mirror fields and modify the transition probability due to gradients in the magnetic or electric field [29]. The search described in this paper assumes these η_B and η_E couplings are absent.

Although Eq. (3) would generically describe the (n, n') system, some simplifying assumptions can be made. In this work we assume that $V' = 0$, $W' = 0$, and $\mu_n = \mu_{n'}$ and simplify the 4×4 matrix Hamiltonian

$$\mathcal{H} = \begin{pmatrix} U - iW & \epsilon_{nn'} \\ \epsilon_{nn'} & 0 \end{pmatrix}, \quad (4)$$

where U includes only real numbers,

$$U = V - \Delta m \pm \mu_n \vec{\sigma} \cdot (\vec{B} - \vec{B}'). \quad (5)$$

Here, the small mass difference $\Delta m = m_{n'} - m_n$ is due to spontaneous symmetry breaking between the ordinary and mirror sectors. The value of Δm can be either positive or negative, where by convention a positive sign for Δm corresponds to $m_{n'} > m_n$. This simplification is possible for either polarization of neutron in an unpolarized neutron beam if the transverse component of the magnetic field does not produce a spin rotation.

Oscillations are driven by the off-diagonal mass mixing term $\epsilon_{nn'}$. The real diagonal elements of Eq. (4), if larger than $\epsilon_{nn'}$, can suppress oscillations between n and n' [28]. Thus, an unknown Δm , $\vec{\mu}_{n'} \cdot \vec{B}'$ or any other environmental potential that is different for n and n' can reduce the oscillations induced by $\epsilon_{nn'}$. However, because these components could have different signs, it is possible to compensate for these competing potentials. The laboratory magnetic field $\vec{B}$ or the material optical potential V can be tuned such that the sum of potentials vanishes, allowing oscillations to occur. Oscillations are maximal when both diagonal terms in Eq. (4) are zero. The evolution of the (n, n') system depends upon the total difference in energy between the n and n' ,

$$\Delta E = U - iW, \quad (6)$$

where the imaginary part of the optical potential W both causes attenuation of the (n, n') system and affects the frequency of oscillations. The solution of the Schrödinger equation describing oscillating (n, n') in absorptive materials with a non-Hermitian Hamiltonian can be found in [27].

In vacuum with $V = 0$ and $W = 0$, and in the absence of fields that are different for n and n' such as the magnetic field $\vec{B}$, the time averaged probability of $n \rightarrow n'$ oscillation at small θ_0 becomes [30]

$$P_{nn'} = \frac{2\epsilon_{nn'}^2}{\Delta m^2 + 4\epsilon_{nn'}^2} = \frac{1}{2} \sin^2 2\theta_0 \simeq 2\theta_0^2 \simeq 2 \left(\frac{\epsilon_{nn'}}{\Delta m} \right)^2. \quad (7)$$

Thus, the probability $P_{nn'}$ in vacuum is determined by two independent parameters $\epsilon_{nn'}$ and Δm . The vacuum mixing angle θ_0 defined by

$$\tan 2\theta_0 \equiv 2\epsilon_{nn'}/\Delta m \quad (8)$$

serves as a rotation angle for diagonalization of the Hamiltonian in Eq. (4) in vacuum. When the angle θ_0 is small, it becomes a convenient parameter for calculating the transformation probability $P_{nn'}$. Δm and θ_0 provide an alternative set of two independent parameters describing oscillation in a vacuum. Experimental limits below are presented in terms of the vacuum oscillation parameters Δm and θ_0 . Extensions to this to account for different material and magnetic potentials have been previously reviewed in [27], where the rotation angle θ_B that diagonalizes the general Hamiltonian of Eq. (4) can be written as

$$\tan 2\theta_B = \frac{\tan 2\theta_0}{(V - \Delta m \pm |\mu_n B| - iW)/\Delta m}. \quad (9)$$

A (n, n') system with a nonzero Δm traveling through a nonuniform magnetic field passes through a dynamic resonance at $U = V \pm |\Delta m| \pm |\mu_n B| = 0$, when the mixing angle approaches its maximum value at $\theta_B = 45^\circ$. This is a classical case of the Landau-Zener (LZ) nonadiabatic “level-crossing” transition [31,32], which enhances the transformation $n \rightarrow n'$ in a similar manner to the solar flavor neutrino transitions [33]. In the case of a beam of n passing through a magnet with peak field $B_{\max}$ and where $\Delta m < |\mu_n B_{\max}|$, these LZ transitions occur at least twice; one level crossing occurs at the entrance of the magnet and one level crossing occurs at the exit of the magnet, when Δm is compensated by $\mu_n B$. If the beam of neutrons is unpolarized, the polarity of the magnetic field does not matter, as half of the neutrons will still undergo the LZ transition. The amplitude of these transitions will depend on the initial phase of the oscillating (n, n') system, the mixing $\epsilon_{nn'}$, the local gradient of magnetic field $\vec{B}$, and the velocity of the neutron v as it passes through the level crossing. In the case where $\Delta m > |\mu_n B_{\max}|$, the level crossing does not take place and the transformation occurs adiabatically at $\theta_0 < 45^\circ$. In this case, the transformation probability is suppressed by $\Delta E = \Delta m - |\mu_n B_{\max}|$ inside the magnetic field region and returns to the initial suppression defined by $\Delta E = \Delta m$ outside.

Analytic calculations of the evolution of the wave function using the Schrödinger equation are difficult in the general case due to the requirement of matching the wave functions and their derivatives at both surfaces of the absorber as well as in the detector. Numeric calculations using density matrix evolution are free from these difficulties. The density matrix evolution equation or Liouville–von Neumann equation [34] is a more general way of describing the interaction of a quantum system with its

environment without using a wave function and is equivalent to the Schrödinger equation in case of a Hermitian Hamiltonian. Transmission probability calculations in this paper used the density matrix evolution equation with a Hamiltonian given by Eqs. (4) and (5) under the assumption of $B' = 0$,

$$\frac{d\hat{\rho}}{dt} = -i\hat{\mathcal{H}}\hat{\rho} + i\hat{\rho}\hat{\mathcal{H}}^\dagger, \quad (10)$$

where $\hat{\rho}(t) = |\Psi(t)\rangle\langle\Psi(t)|$ is a 2×2 density matrix [assuming the (n, n') state is not polarized]. The diagonal elements of this matrix, $\rho_{11}(t)$ and $\rho_{22}(t)$, represent the probability of the oscillating system being found in either a pure n or n' state, respectively. The density matrix is Hermitian such that the off-diagonal elements satisfy $\rho_{12}^* = \rho_{21}$. These latter elements contain information about the phases of oscillation.

The Liouville–von Neumann equation (10) does not treat decoherence effects, and thus, it does not account for the reset of oscillation phases when the (n, n') system scatters on nuclei in materials. For experiments with a cold neutron beam, and for absorber layers that have a relatively small scattering probability, decoherence effects are nonessential since they involve only the neutron component of the mixed (n, n') state and thus behave similar to absorption by removing neutrons from the beam. More generally, however, instead of Eq. (10) one can use the Lindblad equation for the density matrix evolution that includes the macroscopic effect of scattering-induced decoherence [34]. This approach has been used, e.g., by the STEREO Collaboration for the calculation of the probability of transformation of “hidden” neutrons produced from neutrons in a research nuclear reactor through a thick layer of heavy water where the number of elastic collisions with the material is large [35].

C. Current limits on n to mirror n' transformation

The existence of neutron–mirror neutron mixing would cause observable consequences that can be probed in the laboratory. Astrophysical constraints on (n, n') mixing come from mass losses in pulsars and the temperatures of neutron stars [36–39]. New heavy particles that would contribute to the mechanism of $n \rightarrow n'$ transformation have not been observed at the Large Hadron Collider. The case of large Δm or ΔE has also been considered [23,40,41]. Neutron–mirror neutron mixing would result in unexplained neutron disappearance in terrestrial cold and ultra-cold neutron measurements.

For larger Δm , constraints can be obtained from observation of neutron propagation in weakly absorbing materials. The angle θ_0 [Eq. (8)] has a maximum value of $\theta_0 = 45^\circ$, which can be reached for $\Delta m = 0$ or for finite Δm if $\epsilon_{nn'} \gg \Delta m$. The time averaged probability of $(n \rightarrow n')$ transformation in Eq. (7) is $\sin^2 2\theta_0/2$, which

changes from 0.02 to 0.5 for any angle $\theta_0 > 0.1$. Neutron disappearance at this level would be observable, for example, for neutrons propagating in the heavy water reflector surrounding the compact central core in research reactors [42]. Every elastic collision would lead to the collapse of the (n, n') system, leading to the loss of neutrons with probability $> 2\%$. Because the radial neutron flux distribution calculated with MCNP agrees with the measured flux [42], angles $\theta_0 > 0.1$ are excluded for all values of Δm .

Experimental limits on the mirror matter model rely upon specific assumptions about the nature of Δm and $\vec{B}'$ [43] (and possibly on the existence of a neutron transition magnetic moment [29]). The most stringent transition limits, $\tau_{nn'} > 448$ s (90% C.L.), come from the assumption of the perfectly degenerate case, with $\Delta m = 0$ and $\vec{B}' = 0$ and for the laboratory magnetic field $\vec{B}$ close to zero [44].

In other searches with ultracold neutrons (UCNs), the presence of mirror magnetic field $\vec{B}'$ with unknown magnitude and direction was considered [45–47]. A reanalysis of those previous experiments reported a result consistent with $n \rightarrow n'$ losses for nonzero $\vec{B}'$ [45,46]. Subsequent dedicated searches for $n \rightarrow n'$ utilized repurposed experiments to search for the neutron electric dipole moment and have excluded most of these claimed signals [46–48]. More UCN storage search experiments are still in progress [49,50]. In addition to using UCN storage, further $n \rightarrow n'$ disappearance searches have been performed using beams of UCNs passing through a guide with a tunable magnetic field [51], with results obtained using the GADGET UCN detector at the ILL [52].

Neutron regeneration experiments, which search for the double transition $n \rightarrow n' \rightarrow n$, have been discussed in [53,54]. Neutrons that disappear before passing through an absorber would be regenerated on the other side, analogous to similar experiments using photons through a wall to search for exotic electromagnetic couplings [55].

Searches for sterile hidden neutrons n' performed at nuclear research reactors [35,56] have an advantage of using very large initial number of neutrons for conversion to n' . Further searches for such processes have been proposed for a higher flux source with a lower background detector [57]. The effect of neutron regeneration would be seen in a detector as a n counting rate signal above the background correlated with the operation of the source. With detailed accounting of the initial neutron flux of the source, as well as the materials or magnetic fields responsible for the (n, n') conversion, the observed n signal can be translated into a probability of $(n \rightarrow n')$ oscillation.

The theoretical $n \rightarrow n'$ model with $\Delta m \neq 0$ probed in this paper was originally proposed in [30] as a potential explanation for the “neutron lifetime discrepancy.” This discrepancy and proposed searches for physics beyond the Standard Model have been reviewed in [58].

An experimental probe of this effect was proposed at Oak Ridge National Laboratory (ORNL), utilizing the Spallation Neutron Source (SNS) and the High Flux Isotope Reactor (HFIR) [59,60]. A first search performed at SNS [61] excluded $n \rightarrow n'$ transitions as an explanation for the neutron lifetime discrepancy. This paper improves upon the results of this experiment to further limit the observed amount of neutron–mirror neutron mixing.

II. EXPERIMENTAL APPROACH

The measurement reported here was performed using the cold neutron regeneration technique at the Magnetism Reflectometer (MagRef) instrument at the SNS at ORNL. This measurement implemented several improvements over the previous search reported in [61], including higher neutron intensity, improved intensity determination, longer experiment running time, more sensitive choice of neutron absorber material, and reduced backgrounds. A diagram of the MagRef beamline configuration in this experiment can be seen in Fig. 1. The major components of the experiment include the neutron source; beamline components including defining apertures, optional attenuators, and detector; and at the heart of the experiment, the magnet with neutron-absorbing beam catcher.

A. Neutron source

The SNS produces H^- ions that are accelerated by the linear accelerator to 1 GeV with a nominal power of 1.4 MW at the time of the measurement [62]. The accelerated H^- beam is stripped of electrons by a diamond foil and then collected and intensified in the accumulator ring. The protons are then extracted in 700 ns long bunches with a repetition rate of 60 Hz which bombard a liquid mercury target to produce fast neutrons by spallation [63]. The charge in each proton bunch is measured by an integrating current beam monitor with an accuracy of 3% [64].

The neutrons viewed by the MagRef beamline [65,66] are moderated by a 20 K liquid hydrogen moderator [67]. Neutrons are emitted with a timescale of tens of microseconds and transported via a curved guide over a distance of approximately 16 m to the experimental area. A system of beam choppers allows the passage of neutrons only within a user-selected wavelength range, in this case from 2.2 to 5.1 Å. The neutrons then pass through the 5 m long experimental area before reaching the detector. The time at which they complete this flight, referenced to the clock signal corresponding to the arrival of protons on the spallation target, provides a time of flight (TOF) estimate for each neutron hitting the detector. A typical TOF spectrum of the neutron beam is shown in Fig. 2. Determination of the absolute neutron intensity is discussed in the Results section. The neutrons passed by the chopper system arrive at the detector position at times ranging from 12 to 27 ms following their emission from the moderator surface. The analysis discussed in this work removed

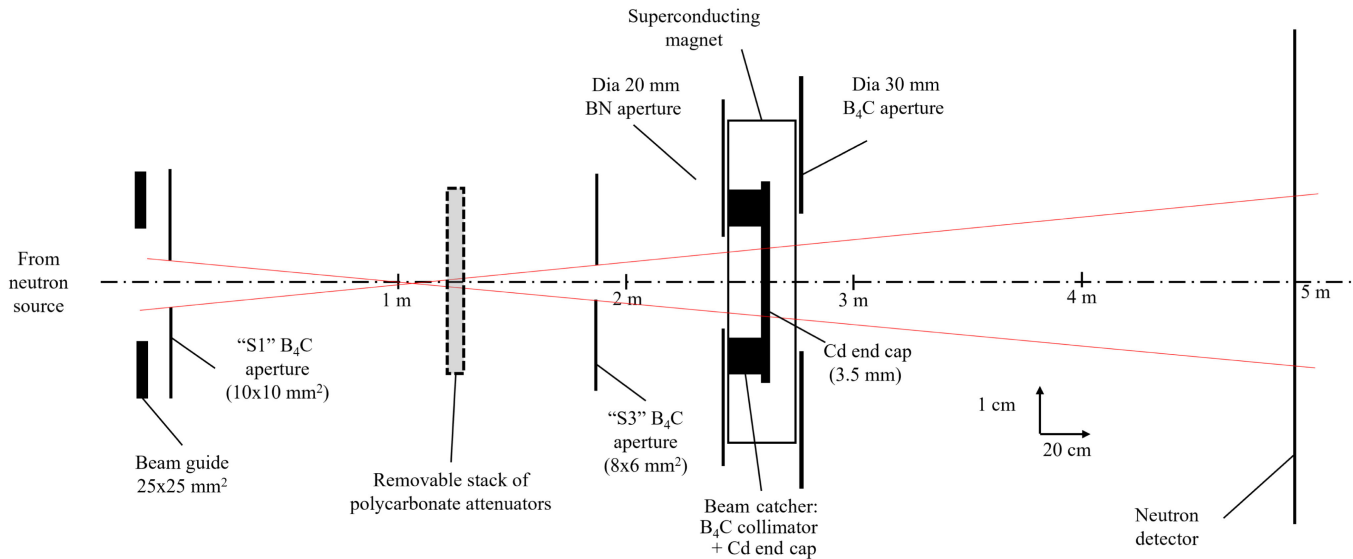


FIG. 1. Diagram of the downstream portion of the MagRef beamline with the cadmium (Cd) absorber in place. Components impacting beam divergence include the exit of the beam guide, the “S1” boron carbide (B_4C) aperture, the “S3” boron carbide aperture, a boron nitride (BN) aperture installed on the upstream entrance to the magnet, a boron carbide collimator with cadmium end cap installed inside the magnet, and a boron carbide aperture at the downstream exit of the magnet. S1 and S3 are part of the MagRef instrument; the remaining apertures and collimators were added for this experiment.

neutrons near the edges of the chopper spectrum, only including neutrons in the interval of TOF between 13.0 and 26.0 ms. This TOF exceeds the time between accelerator frames at 60 Hz, resulting in a “prompt flash” of background at 16666 μ s. This background is excluded from further analysis by ignoring any events arriving in a 50 μ s window around 16666 μ s. At nominal SNS power of 1.4 MW the proton beam charge was stable with an average charge of $Q_1 = 23.32 \mu$ C per proton pulse and a 1σ variation of 0.07 μ C over the three day running period. The mean charge in any single 1 h run was between 23.2 and 23.5 μ C, and the 1σ variation within an hour was also 0.07 μ C per pulse. To avoid the possibility of variation in neutron beam intensity caused by nonlinear moderator performance (see [68]) or from variation in detector dead time consequences, frames with $Q_1 < 23.0 \mu$ C were excluded from our analysis.

The neutron time of flight over a fixed distance can provide the neutron velocity, which in turn can be related to the de Broglie wavelength,

$$\lambda(t) = \frac{h}{p} = \frac{h}{m_n} \frac{t}{L}, \quad (11)$$

where the constant $\frac{h}{m_n}$ is $3.956 \text{ mm} \cdot \text{\AA} \mu\text{s}$, the time of flight t is in microseconds, and the length L is in millimeters. Imperfect knowledge of the flight path length (due to multiple possible trajectories through the guide system) and the neutron emission time (due to the wavelength-dependent emission time dispersion from the moderator) can be calibrated using the Bragg edges of aluminum in

window materials along the neutron beamline. These Bragg edges appear at 2.864, 4.050, and 4.676 $\text{\AA}$ [69] and in Fig. 2 appear as features at 15230, 21430, and 24658 μ s, broadened by the dispersion in emission time of the neutrons from the moderator surface (around 200 μ s) and actual distance through the neutron guide. These indicate an effective flight path L of 20955(11) mm, very close to the nominal 21 m.

B. MagRef beamline and detector

Upon exiting the neutron guide inside the biological shielding surrounding the SNS target monolith, the

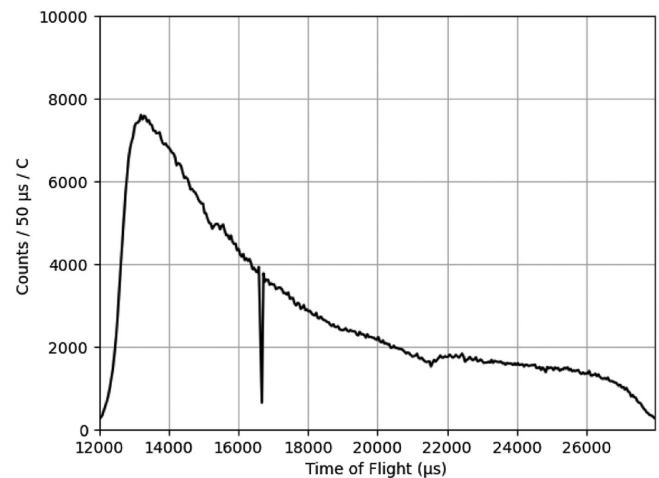


FIG. 2. Time of flight spectrum for neutrons hitting the MagRef detector during a run taken with a small pinhole in the beam. This includes a 50 μ s cut around 16666 μ s to exclude the prompt flash.

neutrons encounter a set of vertical and horizontal B_4C blades (S1 in Fig. 1) serving as an aperture that allows a neutron beam with a maximum permitted extent of $\sim 30 \times 30$ mm to enter the MagRef experimental area. The neutrons travel unimpeded through air nearly 2 m before encountering a second aperture, which defines the divergence of the beam. For this measurement, the upstream aperture (S1) permitted a beam extent of 10×10 mm² and the downstream aperture (S3) permitted a beam extent of 8×6 mm² (horizontal $\times$ vertical).

A set of polycarbonate (PC) $(C_{16}O_3H_{14})_n$ plates stacked together was installed 0.7 m upstream of the downstream aperture (S3) and used to attenuate the beam for an indirect determination of the total neutron intensity, as described in Sec. III. It was removed during all other measurements. The plates were stacked perpendicular to the beam. The nominal “sample position” of the beamline, where in typical material science measurements neutrons are scattered from material samples inside of a magnet, is 0.7 m past the downstream aperture (S3). In the measurements reported here, a 3.5 mm thick absorbing cadmium plate was installed at the sample position. The magnet configuration used for this experiment is described in the next subsection. Further downstream, the neutron beam passes through a shielded beam tube before encountering the detector 2.3 m downstream of the sample position. A final set of vertical and horizontal B_4C blades in front of the detector were positioned to fully enclose the shielded beam tube to reduce scattered neutron backgrounds from the room. Although MagRef has the capability to polarize the neutron beam, these measurements did not use a polarized beam.

The MagRef detector was developed by Brookhaven National Laboratory and uses a ^3He converter to detect neutrons through the reaction $n + ^3\text{He} \rightarrow p + t$ with $> 90\%$ efficiency [70]. The p and t have a short range O(mm) in the nominal 6.8 atm of ^3He plus 2.7 atm propane (buffer gas) in which they produce ionization. The multi-wire proportional counter detects the ions and provides position sensitivity through cathode readout with multinode charge division with spatial resolution of about 1.4 mm. The detection efficiency is not perfectly uniform with position, but the technique reported here is not sensitive to nonuniformities in efficiency. Small drifts in the detector efficiency would happen on the scale of months and thus are negligible over the few days measured here.

An advantage of this detector technology is the ability to discriminate against γ rays, due to the much lower ionization density of γ -induced electrons. This is critical given the use of a fully absorbing cadmium sheet required for this experiment which fully converts the neutron flux at the sample position to an isotropic γ field. The detector is installed on a rotating arm, used to view large scattering angles in other experiments, but was oriented perpendicular to the beam axis for these measurements.

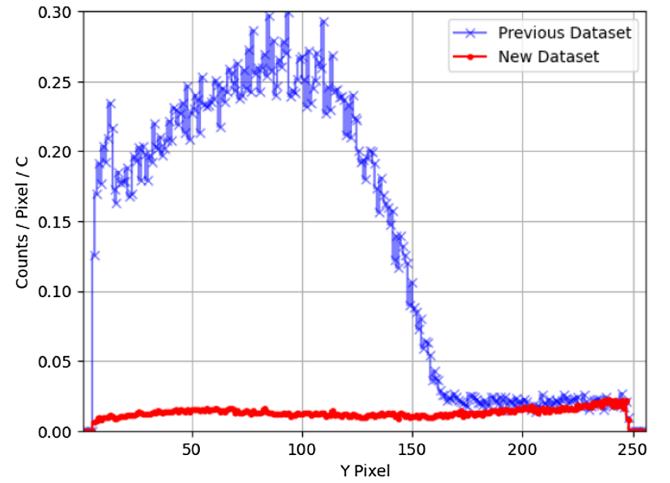


FIG. 3. Background counting rate normalized to proton charge versus vertical position on the detector during the measurement reported in [61] (blue) and after improving shielding against backgrounds for the measurement reported here (red).

Data taken using the detector are represented as an array of 304×256 ($X \times Y$) pixels, each with area 0.7×0.7 mm² [71]. Each single detected neutron is subsequently recorded into a data acquisition system which includes a GPS-based time tag, the (X, Y) position of the pixel in the detector, the time of flight, and the proton charge for the SNS pulse that produced the neutron.

The ambient background counting rate was a significant limiting factor in the previous measurement reported in [61]. For this measurement, the detector was surrounded by additional shielding made of boron carbide and $\sim 5\%$ (by mass) borated polyethylene. The downstream neutron beam guide was brought closer to the detector apertures. This removed a line of sight to neutrons scattering from the granite and concrete floor. The background counting rate was reduced by a factor of 10 and a significant asymmetry in the vertical direction was eliminated (Fig. 3). Measurements performed with the neighboring beamline shutter opened and closed confirmed that no additional background was observed due to external sources. The magnetic fields at the detector were too small (< 1 mT when the magnet was on) to affect the detector performance.

C. Magnet and beam catcher

The central feature of the experimental approach is the neutron evolution through the magnetic and material optical potentials that create the disappearance and regeneration effect, depicted in Fig. 4. Example neutron transmission probabilities through the magnetic field are shown. With no magnetic field, neutrons are fully absorbed in the material, and none are detected by the MagRef detector. In the presence of the magnetic field, the LZ transition enhances the probability that the neutron is “observed” in the mirror neutron state at the absorber, and another LZ

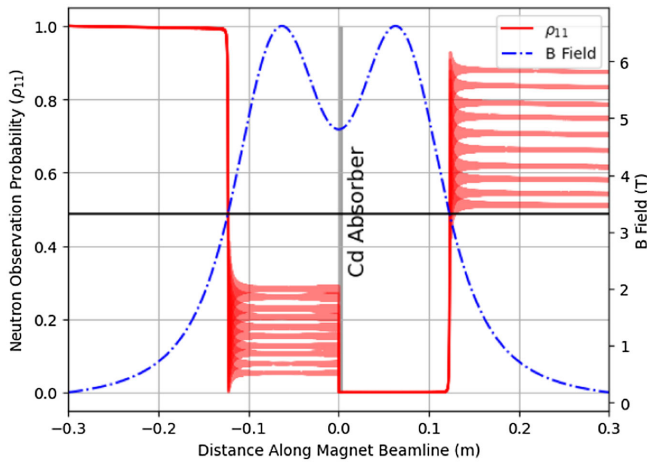


FIG. 4. Example of a calculated evolution of the (n, n') system through the MagRef field and absorber for $\Delta m = 200$ neV, $\theta_0 = 5 \times 10^{-3}$, and using 10 wavelengths sampled from the distribution in Fig. 2. The black horizontal line indicates the magnetic field that would correspond to Δm . The square of the neutron wave function as a probability of neutron observation is shown for each velocity sampled from the distribution. The effect is calculated for an absorber of 3.5 mm Cd (gray vertical line) as used in this experiment.

transition results in a regeneration of neutrons upon exiting the magnet.

For this experiment, a Cryomagnetics, Inc. superconducting split pair magnet was installed in the MagRef beamline. The magnet is 30 cm long and composed of two solenoids, such that the magnetic field has a double peak [72]. The dip in the center of the magnetic field is caused by a gap between the solenoidal coils intended for samples to be inserted for exposure to the neutron beam. When fully ramped, the magnetic field in the center of either of these two solenoids (at the local maxima at 6.3 cm upstream and downstream of magnet's center) is 6.6 T, while the field in the dip at the center of the magnet is 5 T. The magnet vendor provided the calculated field profile for radii up to 5 cm and for ± 30 cm along the magnet axis. The magnetic field was measured on axis with a group 3 Hall probe, calibrated with 0.1% precision, and was found to be between 1% and 2% higher than the provided field profile. The impact of this difference on the sensitivity is small. The measured stray fields outside the magnet were also provided. Some components of the MagRef beamline increased the magnetic field locally. The ambient magnetic field at the beamline entrance at slit S1 was measured to be 1 mT and the field at the detector was less than 0.1 mT, which sets the limits of our sensitivity. From simulation, the experiment sensitivity was not significantly impacted by the shape of the magnetic field in the beamline far from the magnet, as long as the neutron originated in a lower magnetic field.

The magnetic field profile for the major component of magnetic field can be seen in Fig. 4. The oscillation probability of the (n, n') state depends on the magnetic

field, the orientation of the neutron spin, the neutron velocity, and the parameters of the mirror matter model. The search was conducted with the center of the magnetic field set to ± 4.7 and ± 2.35 T, as well as a background measurement with no nominal magnetic field. When the coils had no current, the maximum residual field inside of the magnet was < 3 mT. This would be equivalent to a $\sim 0.2\%$ uncertainty on the field at full strength or a $\sim 0.3\%$ uncertainty on the potential of the cadmium beamstop during background runs. The beam is unpolarized, and so the oscillation probability is independent of the polarity of the magnetic field. During steady-state operation, the current supplied to the magnet was monitored and the fluctuations in the coil currents ($< 1\%$) were negligible for this measurement.

Inside the magnet, a 15 cm long and 25.4 mm inner diameter neutron-absorbing cylinder (B_4C) with an absorbing end cap (cadmium) was installed in the beam path to remove neutrons while allowing n' transmission, serving as a beam catcher. A tightly fitting 3D-printed “tough polylactic acid” support was used to axially center the beam catcher in the magnet, with a slight (< 1 mm) vertical displacement due to gravity. The end cap was installed centered inside the magnet with a positioning precision of about 1 mm. The neutron absorber end cap was changed from 32 mm of B_4C as used in the previous setup [61] to 3.5 mm of Cd for better sensitivity as described later.

The intent of the beam catcher is that the neutrons enter the tube unimpeded and are absorbed by the end cap, but if neutrons backscatter they can be absorbed by the walls instead of potentially increasing backgrounds to the detector. However, intensity measurements were performed without the beam catcher in place. It was important to ensure that neutrons were not absorbed by the beam catcher walls before reaching the end cap, which would have resulted in an overestimate of the neutron flux (and thus sensitivity) during the experiment. A neutron camera placed in the magnet position was used to estimate the optimal upstream aperture settings in order to maximize the intensity through the beam catcher without any loss or scattering of neutrons in the beam catcher walls due to misalignment or beam divergence. The neutron beam divergence was checked by comparing the neutron beam extent measured with this neutron camera to the extent measured by the main MagRef detector. The magnet was installed on a two-dimensional linear and rotational stage, and the position and vertical axis angle were scanned to find the peak count rate on the detector, such that the neutron beam was roughly centered. To ensure the neutron beam did not encounter the beam catcher walls, a 20 mm boron nitride aperture was installed at the front face of the magnet, which was smaller than the opening of the beam catcher. A 30 mm boron carbide aperture was installed on the downstream face of the magnet to prevent scattering outside of the beam catcher. The beam catcher and

apertures were centered on the magnet with an accuracy of <1 mm. A comparison of the neutron beam position distribution as measured by the MagRef detector using different apertures demonstrated that reducing the aperture diameter had no measurable impact on intensity. Therefore, the extent of the neutron beam was centered inside the beam catcher and the beam extent was smaller than the acceptance. Also, additional background induced by scattering on the beam catcher walls was shown to be $<1\%$ of the total beam intensity, sufficient for this measurement. Simulation studies confirmed that the neutron beam divergence did not significantly impact the sensitivity of this measurement.

III. RESULTS

A. Intensity measurement analysis

The probability of the $n \rightarrow n'$ effect probed here must be determined for each wavelength present in the beam. Thus, the absolute spectral intensity of the cold neutron beam must be determined. The total neutron intensity reaching the MagRef detector varies with SNS proton beam charge, the conditions of the moderator, and the neutron beamline settings. While the proton charge is monitored and recorded for each SNS 60 Hz pulse, the moderator condition can vary slowly with an impact on spectral intensity that is not tracked. Further, beamline settings such as aperture positions and sample locations are nominal only. Therefore, the spectral intensity of the neutron beam is not necessarily constant and must be determined for each experiment configuration separately, with a correction for the proton beam charge. As the MagRef beamline was not equipped with any additional device for intensity measurement, the spectral intensity was determined indirectly using the MagRef detector.

The full beam intensity needed for the experiment was too high for the MagRef detector to be measured without saturation, excess noise, or damage to the detector. The determination of the intensity of the neutron beam was therefore performed using an indirect approach, by measuring the intensity of the beam after passing through scattering attenuators with different thicknesses. The attenuator was composed of a variable number of PC plates with equal thickness. The thickness of each plate was measured to be 0.127 cm and the density was 1.195 g/cm³. Six configurations using stacks of 18, 19, 20, 21, 22, and 24 PC plates were positioned in the beam 3 m upstream from the detector for 20 min of data collection in each configuration. The maximum detector counting rate during the intensity calibration was ~ 2500 cps in the beam region of interest with 18 PC plates stacked together. A typical beam profile with 18 PC plates can be seen as a 2D plot in Fig. 5.

The MagRef detector counting rates (per coulomb of the proton beam charge) were measured for different configurations with different total PC thickness, then extrapolated

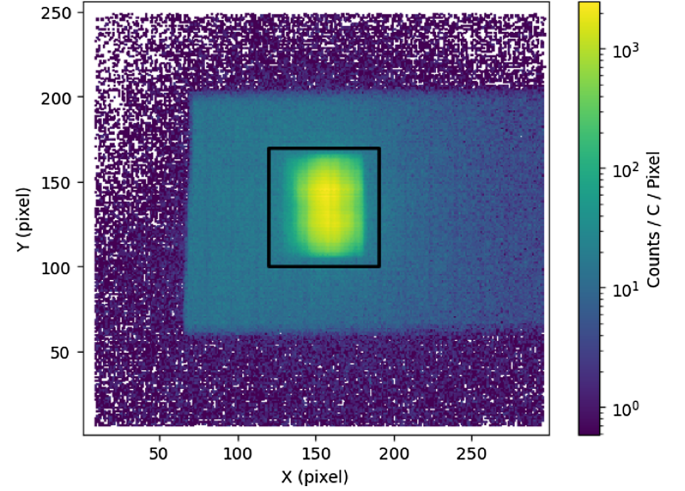


FIG. 5. 2D plot illustrating intensity calibration run taken with a stack of 18 PC plates upstream of the MagRef detector. The attenuated primary neutron beam can be clearly seen as the peak inside the black square indicating the region of interest. The shielding around the detector causes a sharp falloff in the background counting rate.

to the counting rate of zero PC thickness to determine the original beam intensity. The neutron beam attenuation in the PC material is mostly due to elastic scattering on hydrogen. When combined with attenuation of the non-scattered beam and some small absorption, this scattering generated background around the primary beam peak in the detector.

The primary beam was centered at pixels $(X, Y) = (156, 136)$ of the MagRef detector with full width half maximum in X and Y of 24 and 41 pixels, respectively, as shown in Fig. 5. The beam spot forms a peak that corresponds to neutrons passing through the PC attenuator stack without interaction. The direct beam-on area in the figure was used to determine a region of interest (ROI) of 70×70 pixels, which was used in searching for the effect of $n \rightarrow n'$. Outside the ROI in the detector, there is a background generated mostly by the neutrons scattered from the PC attenuator plates. A shielding box around the beamline causes additional absorption, which leads to a falloff in the background count rate above pixel $Y = 200$, below pixel $Y = 70$, and on the left side of the detector below pixel $X \sim 75$. This falloff can be seen in Fig. 5. The region within these cutoffs is defined as the region of background (ROB). The background present in the ROI needs to be subtracted to determine the attenuated fraction of the beam that does not interact with the PC material.

Two components in the measured background were identified. One component was almost uniform in the ROB of the detector as can be seen in Fig. 5. This component can be described as a constant at some initial point with two slopes defining the background plane. The second component forms a scattering bump around the

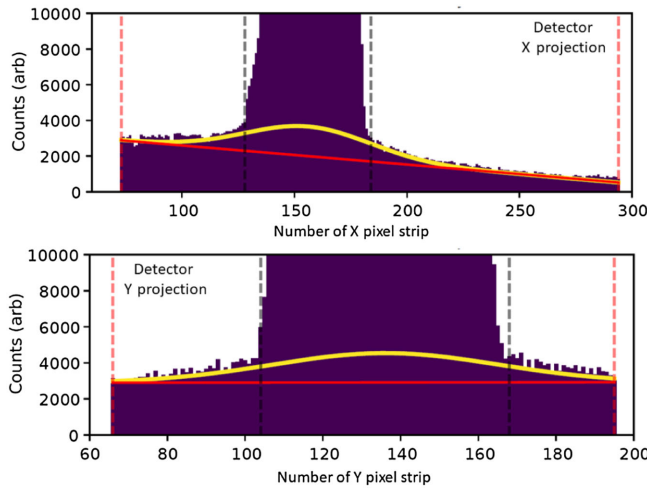


FIG. 6. Example of a 2D background fit with four free parameters projected in X and Y for 18 PC attenuators (see text). Red vertical dashed lines correspond to the region of background and black vertical dashed lines to the region of peak (ROP), which is excluded from the background fit. The solid lines show the contribution of the “plane” background (red) and the “Gaussian bump” background (yellow). Combined statistical and systematic uncertainties of the subtractable background determination are within the width of the Gaussian (yellow) line.

beam peak center. The shape of this bump is determined by the acceptance of the rectangular aperture S3 and the round 20 mm diameter boron nitride apertures shown in Fig. 1. The bump is described by a 2D Gaussian around the center of the beam peak with three parameters: the height at the peak center and the two Gaussian widths σ_x and σ_y . The description of these two components of background was confirmed by a PHITS [73] simulation using Evaluated Nuclear Data File (ENDF)/B-VIII.0 data [74] for a cold neutron beam scattering from PC material¹ in the detection geometry shown in Fig. 1.

The background observed in each PC measurement in the 2D matrix of pixels was fitted to a 2D function combining the plane background with three parameters and the Gaussian bump background described with another three parameters. For this result, it was not essential to perfectly parametrize the more complicated details of the detector response, as long as the general features of the background’s position dependence was captured. The χ^2 metric was used as the figure of merit to select the most appropriate background model. The region of the fit for minimization of χ^2 included the ROB with the exclusion of a region containing the beam peak (ROP), with statistical uncertainties given by $\sqrt{N}$ for each individual pixel. The ROP used for the background fit was smaller than the ROI

described above. A typical example of a background fit in X and Y projections for 18 PC is shown in Fig. 6. In this figure, the χ^2 was 24916 for 20238 degrees of freedom (d.o.f.), leading to a reduced $\chi^2/\text{d.o.f.}$ of 1.23.

The intensity for each of the six PC measurements was obtained by subtracting the fitted background function, integrated over all pixels in the ROI, from the total measured counts in the ROI. The subtractable background and the functional fit parameters were monotonically dependent on the number of PC plates and the subtractable background varied from $\sim 5\%$ for 18 PC plates to $\sim 18\%$ for 24 PC plates. The Gaussian widths, σ_x and σ_y , were found to be consistent across all numbers of PC plates due to being determined solely by the acceptance geometry. The parameters σ_x and σ_y were fixed to their best fitted values to be the same for all PC runs. Thus, the number of free parameters for the background description is effectively reduced to 4.

For each time-of-flight bin, a nonparalyzable dead time correction was applied by calculating the factor $1/(1 - \tau \cdot i)$, where i represents the intensity per bin in units $n \cdot s^{-1}$ similar to Fig. 2, and the dead time $\tau = 4 \mu s$. This factor then was averaged over the frame intensity per second and used to scale the beam intensity. The maximum observed dead time correction, during the highest-rate attenuation run with 18 PC plates, was 2%. A normalization for the variation of the SNS proton charge per run was also implemented to express measured intensities in counts per coulomb.

The uncertainties of the measured intensities included statistical error from the total counts in the ROI and the systematic error of the fitted background subtraction. The latter was estimated from each of the six fit covariance matrices by generating 10^6 random samples of the background in the ROI with the background parameters randomly taken from the multinormal distribution [76] with the given fit covariance matrix. The standard deviation of the 10^6 generated background values for every PC point was taken as a systematic error of the subtractable background in ROI. The relative statistical error for the six measured points increases monotonically with the number of PC plates (18–24) from 0.1% to 0.37% and the relative systematic error increases monotonically from 0.1% to 0.86%.

The six intensities measured with different number of PC plates were fitted to zero PC thickness with the function $I(k) = I(0) \cdot \eta^k$ with two fit parameters, where $I(0)$ is beam intensity for $k = 0$ PC plates, and η is the average beam attenuation factor per single plate. The fitted values of η from the attenuation data will be denoted as η_{exp} . Statistical and systematic errors were added in quadrature as the errors used for the intensities in the fit procedure. The results of the fit of six points extrapolated to the intensity with no attenuation for this background scheme resulted in the following values of parameters: $I(0) = (2.564 \pm 0.029) \times 10^9$ counts/C

¹The scattering kernel “h-luci.40t” for hydrogen bonded in PMMA that was used in PHITS came from the JAEA’s MCNP-formatted JENDL-5 ACE library [75], which is derived from the ENDF/B-VIII.0 data.

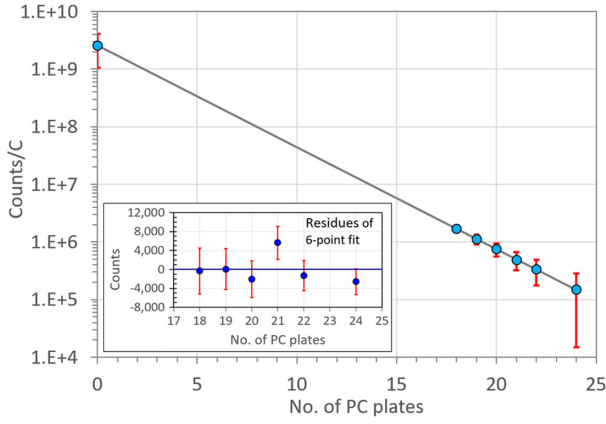


FIG. 7. The extrapolation of the fit of measured intensities to zero PC plate thickness. The six measured point errors are multiplied by factor 50 and the error $I(0)$ by factor 20. Insert: residues for the fit with uncertainties scaled up as described in the text.

and $\eta_{\text{exp}} = 0.6659 \pm 0.0003$. The $\chi^2/\text{d.o.f.}$ for these fits was relatively large at 10.2. The intensity found with 21 PC plates contributes the most to the $\chi^2/\text{d.o.f.}$ and repeating the fit procedure without this run drops the $\chi^2/\text{d.o.f.}$ to 1.34. This could indicate an unaccounted systematic in the measurements. We therefore scale up the uncertainties using the Particle Data Group prescription for the scale factor [77],

$$I(0) = (2.564 \pm 0.075) \times 10^9 \text{ counts/C},$$

$$\eta_{\text{exp}} = 0.6659 \pm 0.0010. \quad (12)$$

Figure 7 shows the extrapolation of the six-point fit to zero PC plate thickness. The total errors shown in the plot for the six measured points are multiplied by a factor 100, and the error of the parameter $I(0)$ at zero thickness is multiplied by a factor 20 for visibility. The insert in Fig. 7 shows the residues for the fit using actual errors. The total neutron beam intensity for the measurements reported here were somewhat higher than in the previous search [61] due to the difference in slit configurations.

A PHITS simulation was used to estimate η by using the scattering kernel for the bound molecular state of the scattering PC material. This simulation used the same attenuation thicknesses for each PC plate. The simulated number of neutrons transmitted to the detector without interaction for different PC thicknesses was fitted to extrapolate to the intensity of zero PC thickness, obtaining $\eta_{\text{sim}} = 0.6746 \pm 0.0009$, which is close to the experimental value of η_{exp} in Eq. (12). The disagreement with the measured η_{exp} is reasonable considering the uncertainty of the exact chemical structure of the PC material used and the absence in the ENDF/B database of the scattering

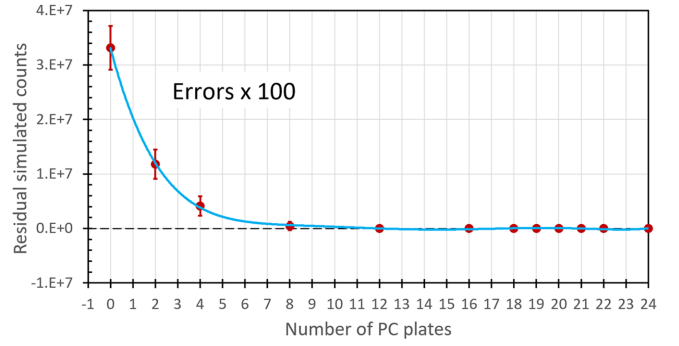


FIG. 8. Residuals of a two-parameter exponential fit of the last six intensity calibrating points corresponding to 18–24 PC attenuating plates, simulated with PHITS using room temperature PC material. Each point in this simulation used 10^9 initial neutrons, except for the PC 0 simulation which used 10^8 initial neutrons with statistics scaled up by 10 to match the other points. The points used in the exponential fit (18–24 PC plates) are both positive and negative, clustered around 0. The increase of the slope toward zero PC thickness demonstrates thermalization of the cold neutron spectrum, resulting in a required correction to the intensity measurements.

kernel for polycarbonate proper, where only the kernel for hydrogen in PMMA was available and used instead.

Because of the very long extrapolation distance from 18 to 0 PC, the accuracy of the simple exponential behavior was checked in simulation. An effect was observed due to up scattering of the cold beam on the room temperature material of PC plates in the PHITS simulation, by simulating smaller thicknesses of PC plates in the range from 0 to 18 plates. The effect of the increase of the local exponential slope (smaller k) is shown in Fig. 8. In the simulation, the cold spectrum of neutrons scattered from the PC plates was transformed by scattering to a thermal neutron spectrum, which has been previously observed [78]. To account for this effect, a correction was applied to the experimentally determined zero PC intensity. The correction was taken from the PHITS simulation as a ratio of the intensity hitting the detector with zero PC attenuation, to the intensity determined by extrapolating to zero thickness from the set of simulated intensities attenuated by 18–24 PC plates. The correction factor was found as 1.254 ± 0.029 . The same correction factor was applied to the experimentally measured extrapolated to zero PC intensity in Eq. (12), thus arriving at the final value of the intensity estimate in the experiment with relative error 3.8%,

$$I_{\text{final}}(0) = (3.215 \pm 0.121) \times 10^9 \text{ counts/C}. \quad (13)$$

The contributions of the uncertainty in the rethermalization correction factor and measurement uncertainty in determination of the beam intensity are roughly the same.

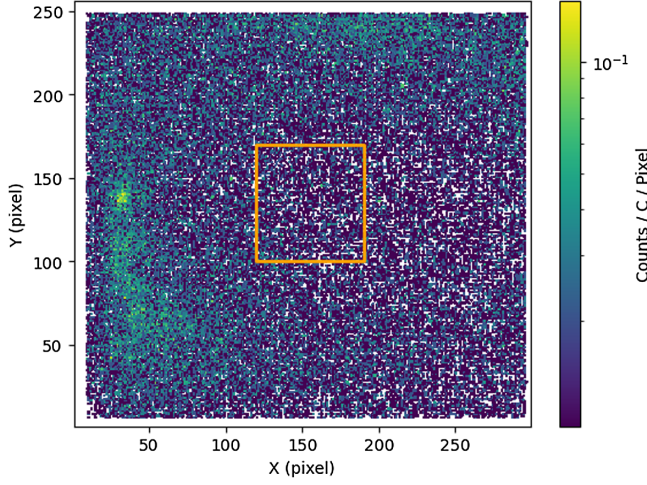


FIG. 9. The sum of all 19 runs taken with the magnetic field off and the cadmium absorber in place. The signal region of interest is outlined in the center of the plot.

B. Search for the effect

To search for or place a limit on the transition regeneration probability for $n \rightarrow n' \rightarrow n$, a set of runs was taken with variable magnetic fields, with the cadmium absorber in the center of the magnet to block the primary neutron beam and the PC attenuators removed. The probability of oscillations into the mirror neutron state and back is enhanced when the energy splitting between the two states is minimized by the magnetic field compensating the mass splitting. The regeneration effect should appear in the ROI of the detector as an excess of the neutron counting rate over the rate of background when magnetic field is present. The ROI was the same as used in the intensity determination. The summed data from the detector with the cadmium absorber in place, formed by the combination of runs taken with no magnetic field at 0 T, are shown in Fig. 9. The measured counting rate at $B = 0$ T was used as a background to the measured signal at the $B = \pm 4.70$ and $B = \pm 2.35$ T nominal magnetic field settings.

As discussed in Sec. II A, the SNS accelerator system delivers submicrosecond long pulses of protons to a spallation target at 60 Hz, inducing a periodic background feature, the so-called prompt flash every $1/60$ of a second. To avoid this background, we placed a veto on events with a

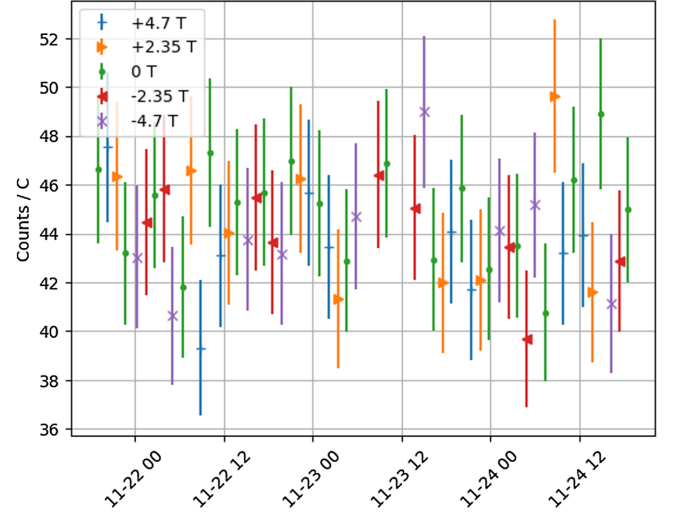


FIG. 10. Measured neutron counts per coulomb of proton beam charge inside the region of interest for each run. Runs are categorized by nominal magnetic field.

time of flight between 16661 and 16711 μ s after the proton pulse signal. Data were collected and organized into runs with constant proton charge, such that the integrated charge on the accelerator target for a given run was 5.1 C. Temporary outages of the accelerator were observed, resulting in longer runs and suppressed rates during the outages. Temporary outages longer than 10 s affected 42% of the typically 1 h long runs. The accelerator pulses were logged in the data and associated with proton pulse charge. During typical operation, the accelerator was stable such that each pulse deposited $23.32 \pm 0.07 \mu$ C into the target. After a temporary outage, the accelerator took some time to ramp up, during which time the neutron intensity delivered from the moderator is expected to be nonlinear as a function of proton charge [68]. To mitigate this effect, accelerator pulses where the proton charge was less than 23 μ C were removed from the analysis.

After these cuts, the neutron counts measured in each individual run were normalized to the accumulated charge for that run. Normalized counts inside the region of interest for individual runs can be seen in Fig. 10. Uncertainties on the raw counts inside the region of interest are assumed to follow a Poisson distribution, and the error bars indicate 1

TABLE I. Combined total signal in ROI region for each magnetic field nominal setting. Uncertainties are statistical and represent 1 statistical deviation.

B field (T)	Total time (s)	Live time (s)	ROI counts	Total charge (C)	Counts/C
+4.70	33147	32559	1987 ± 46	45.6	43.5 ± 1.0
+2.35	32726	32591	2027 ± 45	45.6	44.4 ± 1.0
0.00	81179	68853	4324 ± 66	96.3	44.9 ± 0.7
-2.35	32801	32614	2010 ± 45	45.6	44.1 ± 1.0
-4.70	39014	32612	1999 ± 45	45.6	43.9 ± 1.0

standard deviation. The individual runs were combined to provide an overall signal for different magnetic fields, shown in Table I. From Fig. 10 and Table I, one can see that background measurement with field $B = 0$ T is consistent within error with the measurements with positive and negative values of magnetic field. This signifies that within measurement errors, no $n \rightarrow n' \rightarrow n$ is observed.

C. Limits of Δm and θ_0 parameters of theory

Determination of the limit on $n \rightarrow n' \rightarrow n$ utilizes the Feldman-Cousins method for treatment of rare events [79]. The Feldman-Cousins method selects a likelihood range closest to the most likely one and uses this to determine the 95% confidence limit (C.L.) on the regeneration transition p_{tr} . These limits were calculated assuming Gaussian uncertainties for the signal and backgrounds. As mentioned above, background measurements come from the 0 T data inside the ROI.

A summary of the search for the exotic regeneration effect at different magnetic fields is presented in Table I. Since the neutron beam is not polarized, the positive and negative signs of the magnetic field can be averaged before the subtraction of the zero magnetic field background. The intensity of $(3.215 \pm 0.121) \times 10^9$ counts/C from Eq. (13) is applied to determine the probability of the regeneration effect per initial neutron, averaged over the spectrum of neutron beam velocities. The corresponding Feldman-Cousins 95% confidence level limit is shown in Table II. To relate this measured result of the excluded probability of transmission to the corresponding parameters Δm and θ_0 of the theoretical model [30], simulations of the neutron evolution through the experimental apparatus were performed.

D. Simulations of neutrons in matter and magnetic fields

The probability of $n \rightarrow n'$ transmission through the absorber can be calculated for varying Δm and θ_0 parameters [27]. The density matrix equation (10), containing the Hamiltonian defined in Eq. (4), was used to describe the evolution of the (n, n') system through the 60 cm path shown in Fig. 4, including the magnetic field and cadmium

absorber. For each initial neutron with given velocity v , Δm , and θ_0 , the evolution equation was solved numerically utilizing a custom GPU simulation package in the Julia programming language.

The Liouville–von Neumann equation (10) can be written using the matrix form $\dot{\rho} = \mathbf{M}\rho$, where within a sufficiently small step $dt = vdz$ the matrix $\mathbf{M}$ is constant. The solution to the differential equation can be found with the matrix exponential,

$$\rho(t) = e^{\mathbf{M}t} \rho_0. \quad (14)$$

The matrix exponential is calculated by finding the Jordan decomposition of the matrix $\mathbf{M}$,

$$\mathbf{M} = \mathbf{S} \mathbf{J} \mathbf{S}^{-1}, \quad e^{\mathbf{M}t} = \mathbf{S} e^{\mathbf{J}t} \mathbf{S}^{-1}. \quad (15)$$

Since the density matrix $\hat{\rho}$ of the Liouville–von Neumann equation is Hermitian, Eq. (10) can be rewritten as

$$\begin{pmatrix} \dot{\rho}_{11} \\ \dot{a} \\ \dot{b} \\ \dot{\rho}_{22} \end{pmatrix} = \frac{1}{\hbar} \begin{pmatrix} -2W & 0 & -2\epsilon & 0 \\ 0 & -W & +U & 0 \\ \epsilon & -U & -W & -\epsilon \\ 0 & 0 & 2\epsilon & 0 \end{pmatrix} \begin{pmatrix} \rho_{11} \\ a \\ b \\ \rho_{22} \end{pmatrix}. \quad (16)$$

Equation (16) uses the Hermiticity of $\hat{\rho}$ to relate the off-diagonal components of the density matrix $\rho_{12} = a + ib$ and $\rho_{21} = a - ib$. Equation (15) can then be solved numerically for individual neutron trajectories with a fixed Δm , θ_0 , and velocity v using the matrix described in Eq. (16). This calculation begins with a density matrix at $t = 0$ in the pure neutron state,

$$\hat{\rho} = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}. \quad (17)$$

The potentials in Eq. (16) come from the presence of magnetic fields and material optical potentials. As the magnetic field $\vec{B}$ couples to the spin of the neutron $\vec{\mu}_n$, the calculation must be run for each spin $\pm\mu_n$ state separately and subsequently averaged. Equation (17) defines the initial phase of the oscillating (n, n') system at the time $t = 0$ of the evolution calculation. The averaging over all initial phases is effectively provided by the averaging of the probability (ρ_{11}) of the neutron survival over the spectrum of neutron velocities, related to the time-of-flight spectrum in Fig. 2.

The simulation of the evolution through the variable magnetic field accounts for both neutron polarizations in the case of adiabatic transformation probability where $\Delta m > |\mu B_{\text{max}}|$ as well as the case of LZ transitions at the compensation point where $|\Delta m - \mu B| = 0$. The magnetic field profile of Fig. 4 was provided by the magnet manufacturer and converted into a cubic spline for interpolation. To avoid sharp steps in the magnetic field profile,

TABLE II. Limit on the $n \rightarrow n' \rightarrow n$ regeneration probability per neutron at 95% C.L. with cadmium absorber for two measured magnetic field settings. The ± 4.70 and ± 2.35 T datasets use both polarities with the 0 T data taken as a background. The combined 4.70 and 2.35 dataset uses all constant field-on data, with the 0 T data taken as a background.

B field (T)	Signal ($\times 10^{-10}$)	95% C.L. p_{tr} . ($\times 10^{-10}$)
4.70	-3.7 ± 3.1	3.0
2.35	-2.0 ± 3.1	4.2
4.70 and 2.35	-2.9 ± 2.6	2.6

TABLE III. Material optical potentials used for calculating probability of neutron transmission through an absorber determined using the NIST database [80].

Material	$V + iW$ (eV)
Cd	$5.88 \times 10^{-8} + i \cdot 8.46 \times 10^{-9}$
B ₄ C	$1.99 \times 10^{-7} + i \cdot 6.10 \times 10^{-9}$
Air	$1.20 \times 10^{-10} + i \cdot 5.77 \times 10^{-15}$

this was extended out to a constant field of 50 μ T, accounting for Earth's magnetic field. The mass splitting Δm must be greater than μB at the origin of the simulation in order for Eq. (17) to be valid. The length of the neutron travel must thus be long enough to begin in a region of low field, below 1 mT. The rate of the magnetic field falloff outside the high-field region does not significantly change the transmission probability.

Centered inside the magnetic field is a highly absorbing, 3.5 mm thick, cadmium beamstop, with upstream surface aligned with the midpoint of the magnetic field. This differs from the previous experiment [61] where a 32 mm thick B₄C beamstop was used. The switch to cadmium avoided a strong absorption resonance due to the large optical potential of B₄C (see [27], Table III, and an illustration in Fig. 11) which previously limited the sensitivity between 400 and 600 neV [61]. In addition to the central absorber, the experimental apparatus was located in air at room temperature, which has an additional optical potential. We simulated these materials with both real and imaginary parts of the material optical potentials in Eq. (16) using values in Table III. The addition of air instead of vacuum outside of the absorber provides a $\sim 1\%$ reduction in sensitivity for a given neutron trajectory.

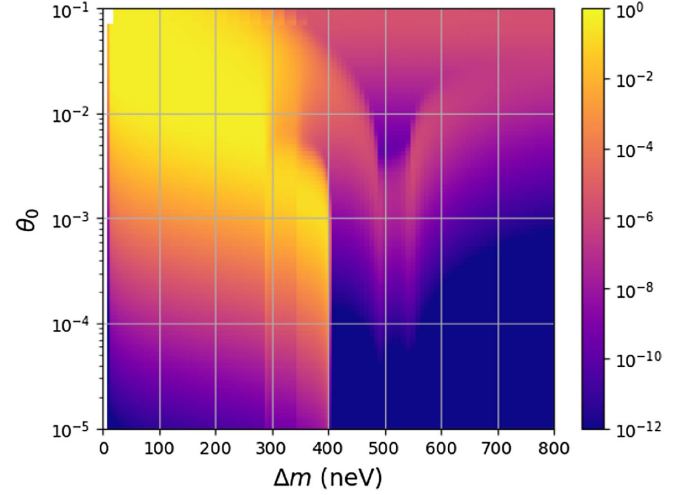


FIG. 11. Simulated probability of regeneration (color scale) through 32 mm of B₄C in a 4.8 T field, the configuration used in [61], as a function of model parameters Δm and θ_0 . Here, Δm is positive. Note the rapidly changing region between 400 and 600 neV due to the Fermi potential of B₄C.

The velocity dependence was accounted for by averaging across 2000 trajectories with fixed velocity, sampled from the velocity profile of neutrons coming from the SNS. This is numerically integrated with steps of constant $z = 30 \mu\text{m}$. Neutrons were simulated along a 2 m total path, with the magnetic field centered in the simulation. The simulated probability for a given neutron trajectory downstream of the magnet at the end of the calculated evolution path has an oscillation behavior similar to that of the (n, n') system in vacuum. For this reason, the probability of transmission was averaged over the final 3 mm (100 steps). The step sizes and number of

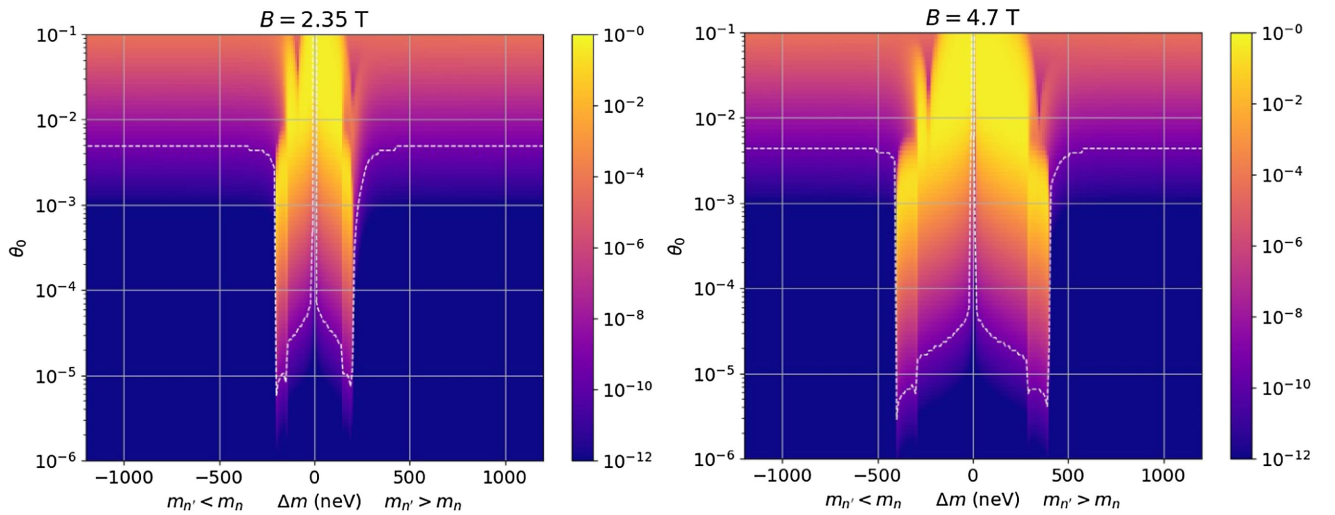


FIG. 12. Simulated probability of regeneration (color scale) through 3.5 mm of cadmium in a 2.35 T (left) peak strength magnetic field and 4.7 T (right) peak strength as a function of model [30] parameters Δm and θ_0 . Area above the dotted line is excluded at 95% C.L. by the sensitivity limit reported in Table II.

trajectories were varied to ensure numerical precision errors were avoided.

Systematic effects, such as positioning of the absorber inside of the magnetic field and the centering of the beam, were studied by running the simulation using an off-axis beam and by changing the location of the cadmium absorber. Shifting the cadmium absorber by ± 1 mm inside the magnet did not change the simulated probability results shown in Fig. 12. Similarly, simulation of neutron evolution using the magnetic field 1 cm off axis did not significantly affect the sensitivity in θ_0 . Simulations of the regeneration probability with both a nominally zero magnetic field and a 3 mT residual field demonstrated that there was no observable reduction in sensitivity due to regeneration during the background measurements.

The results of the simulation described above for a cadmium absorber with the two nominal magnetic fields, 4.70 and 2.35 T, can be seen in Fig. 12. The highest sensitivity to $n \rightarrow n' \rightarrow n$ regeneration can be found near the magnetic field peak. A reduced sensitivity can be seen for higher θ_0 values for smaller $|\Delta m|$ and also when the $|\Delta m|$ exceeds the peak value of magnetic field. The $< 2\%$ higher observed magnetic field when compared to the magnet manufacturer corresponds to a uncertainty in the position of this peak in Fig. 12 by less than one color map pixel (10 neV in Δ_m). For values of $\Delta m > \vec{\mu}_n \cdot \vec{B}_{\max}$, the sensitivity of the experiment is approximately a constant value. For negative mass splittings, the sensitivity only changes by the exact position of the probability void position. At low mass splitting where $\vec{\mu}_n \cdot \vec{B} \gg \Delta m$, the frequency of oscillations becomes less dependent on the specifics of the field profile.

The overall parameter space excluded by this experiment can be found by taking the 95% C.L. $p_{\text{tr.}}$ of Table II and superimposing this on the probability map of the $n \rightarrow n' \rightarrow n$ regeneration as function of parameters Δm and θ_0 for the respective magnetic fields. In calculating these limits, the regeneration effect was assumed to be fully responsible for any counting rate above background in the detector. The ± 4.7 and ± 2.35 T results can be integrated with one another by finding the 95% C.L. of exclusion of the combined probability map. This was done by determining a combined ρ_{11} weighted by the sensitivity of the two results at every simulated point in Δm and θ_0 ,

$$\rho_{11,\text{tot}} = \frac{\rho_{(11;4.70)}/p_{(\text{tr};4.70)}^2 + \rho_{(11;2.35)}/p_{(\text{tr};2.35)}^2}{1/p_{(\text{tr};4.70)}^2 + 1/p_{(\text{tr};2.35)}^2} \quad (18)$$

The Feldman-Cousins procedure can be used to find a new 95% C.L. band for the combined signal from all magnet-on runs. The results can be seen in Fig. 13. The highest sensitivities are at the peak field strengths of the magnet and in the region of high magnetic field enhanced by the Landau-Zener effect.

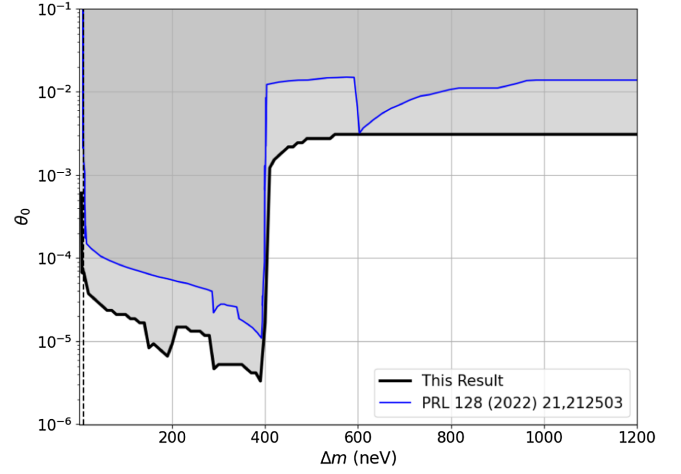


FIG. 13. Comparison between the exclusion limits presented here to the previous work [61] for positive Δm . Note that, in the previous result, the strong Fermi potential absorption resonance in B₄C appears between 400 and 600 neV, leading to less stringent limits than the cadmium absorber.

IV. DISCUSSION

This paper has reported the results of an experiment that provides the most stringent results found thus far for $n \rightarrow n'$ oscillations for the broad region $0.1 \leq |\Delta m| < 10$ neV and competitive results up to 400 neV. Other experiments have used varied techniques in order to probe different mechanisms for (n, n') transitions. These different experiments probe slightly different underlying sources of oscillation and use different analysis methodologies. The experiments can nevertheless be recast in terms of Δm and compared to search across a wide parameter space. We follow a procedure synergistic with the framework for unifying these approaches proposed by [57]. The results from this measurement are compared to other experimental searches in Figs. 14 and 15. Since compensation for Δm comes primarily from a magnetic field, the regeneration technique presented here is relatively insensitive to the sign of Δm . The technique presented here provides competitive limits on neutrons transitioning to mirror neutrons using existing neutron scattering instruments, requiring only a few days of dedicated beam time.

The primary constraints of this experiment come from statistics, understanding of the neutron beam intensity, and understanding of the backgrounds. This technique could be extended for use at higher intensity neutron sources, such as ORNL's HFIR or the European Spallation Source [81,82]. A subsequent search at a reactor source rather than a spallation source would potentially have a more stable neutron flux, providing improvements on the intensity determination. Studies done at higher flux would also require improved understanding of the backgrounds in the detector as well. Extending sensitivity to other energy splittings would require higher strength magnetic fields or better understanding of the low field range.

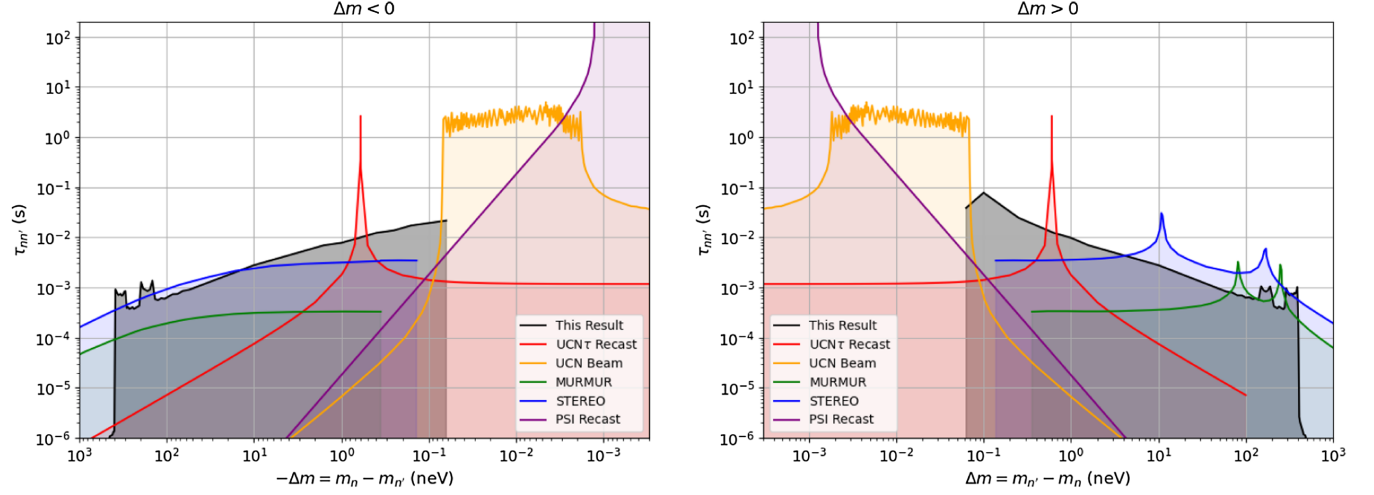


FIG. 14. Limits reported in this paper, overlaid with other searches for sterile neutrons, assuming a unified framework as proposed in [57]. Note that this result is relatively insensitive to the sign of Δ_m , albeit with differences visible near $\Delta_m \sim 0.1$ neV due to the Fermi potential of air. Results from UCN storage at PSI have been recast from searches using searches for a nonzero $\bar{B}'$ [46,47,49]. The limit from UCN τ comes from the nonobservation of anomalous losses, as calculated in [57]. UCN beam results use disappearance in the GADGET detector [52]. The STEREO and MURMUR reactor results are also presented [35,56].

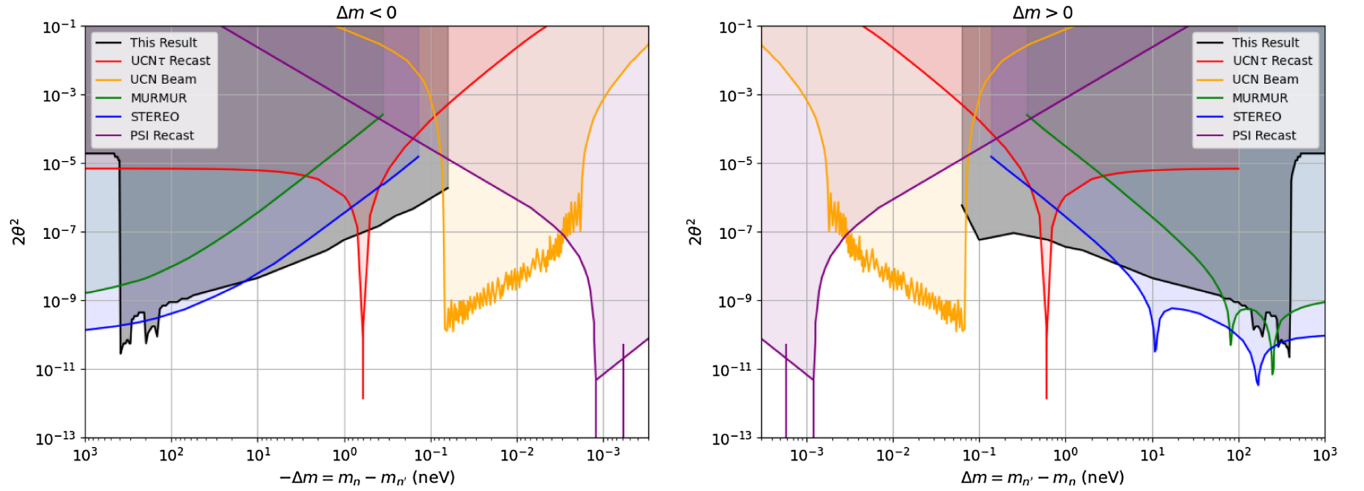


FIG. 15. Same limits as in previous figure but in terms of $2\theta_0^2$, the probability of $n \rightarrow n'$ transformation in vacuum.

The U.S. Department of Energy will provide public access to these results of federally sponsored research in accordance with the DOE Public Access Plan [83].

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- [1] M. Battaglieri *et al.*, in U.S. Cosmic Visions: New Ideas in Dark Matter (2017), [arXiv:1707.04591](#).
- [2] K. K. Boddy, M. Lisanti, S. D. McDermott, N. L. Rodd, C. Weniger, Y. Ali-Haïmoud, M. Buschmann, I. Cholis, D. Croon, A. L. Erickcek *et al.*, *J. High Energy Astrophys.* **35**, 112 (2022).
- [3] S. Tulin and H.-B. Yu, *Phys. Rep.* **730**, 1 (2018).
- [4] S. Adhikari, A. Banerjee, K. K. Boddy, F.-Y. Cyr-Racine, H. Desmond, C. Dvorkin, B. Jain, F. Kahlhoefer, M. Kaplinghat, A. Nierenberg *et al.*, [arXiv:2207.10638](#).
- [5] F.-Y. Cyr-Racine and K. Sigurdson, *Phys. Rev. D* **87**, 103515 (2013).
- [6] T. D. Lee and C. N. Yang, *Phys. Rev.* **106**, 254 (1957).
- [7] I. Y. Kobzarev, L. B. Okun, and I. Y. Pomeranchuk, *Sov. J. Nucl. Phys.* **3**, 837 (1966).
- [8] R. Foot, H. Lew, and R. R. Volkas, *Phys. Lett. B* **272**, 67 (1991).
- [9] Z. Berezhiani, P. Ciarcelluti, D. Comelli, and F. L. Villante, *Int. J. Mod. Phys. D* **14**, 107 (2005).
- [10] Z. Berezhiani, *Int. J. Mod. Phys. A* **19**, 3775 (2004).
- [11] A. Y. Ignatiev and R. R. Volkas, *Phys. Rev. D* **68**, 023518 (2003).
- [12] A. Coc, M. Pospelov, J.-P. Uzan, and E. Vangioni, *Phys. Rev. D* **90**, 1 (2014).
- [13] F.-Y. Cyr-Racine, F. Ge, and L. Knox, *Phys. Rev. Lett.* **128**, 201301 (2022).
- [14] Z. Berezhiani, P. Ciarcelluti, S. Cassisi, and A. Pietrinfermi, *Astropart. Phys.* **24**, 495 (2006).
- [15] R. Ciancarella, F. Pannarale, A. Addazi, and A. Marcianò, *Phys. Dark Universe* **32**, 100796 (2021).
- [16] A. Howe, J. Setford, D. Curtin, and C. D. Matzner, *J. High Energy Phys.* **07** (2022) 059.
- [17] R. Beradze, M. Gogberashvili, and A. S. Sakharov, *Phys. Lett. B* **804**, 135402 (2020).
- [18] N. Aghanim *et al.* (Planck Collaboration), *Astron. Astrophys.* **641**, A6 (2020).
- [19] A. D. Sakharov, *Pis'ma Zh. Eksp. Teor. Fiz.* **5**, 32 (1967).
- [20] L. Bento and Z. Berezhiani, *Phys. Rev. Lett.* **87**, 231304 (2001).
- [21] Z. Berezhiani, *Int. J. Mod. Phys. A* **33**, 1844034 (2018).
- [22] Z. G. Berezhiani and L. Bento, *Phys. Rev. Lett.* **96**, 081801 (2006).
- [23] M. Sarrazin, G. Pignol, J. Lamblin, F. Petit, G. Terwagne, and V. V. Nesvizhevsky, *Phys. Rev. D* **91**, 075013 (2015).
- [24] K. S. Babu *et al.*, in Snowmass 2021 Rare Processes and Precision Frontier (2020), [arXiv:2010.02299](#).
- [25] A. Addazi *et al.*, *J. Phys. G* **48**, 070501 (2021).
- [26] Z. G. Berezhiani, *Eur. Phys. J. C* **81**, 33 (2021).
- [27] Y. Kamyshev, J. Ternullo, L. Varriano, and Z. Berezhiani, *Symmetry* **14**, 230 (2022).
- [28] Z. Berezhiani, *Eur. Phys. J. C* **64**, 421 (2009).
- [29] Z. G. Berezhiani, R. Biondi, Y. Kamyshev, and L. Varriano, *Physics* **1**, 271 (2019).
- [30] Z. Berezhiani, *Eur. Phys. J. C* **79**, 484 (2019).
- [31] L. Landau, *Phys. Z. Sowjetunion* **2**, 46 (1932).
- [32] C. Zener, *Proc. R. Soc. A* **137**, 696 (1932).
- [33] C. W. Kim, W. K. Sze, and S. Nussinov, *Phys. Rev. D* **35**, 4014 (1987).
- [34] H. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Clarendon Press, New York, 2002), p. 107.
- [35] H. Almazán, L. Bernard, A. Blanchet, A. Bonhomme, C. Buck, P. del Amo Sanchez, I. El Atmani, L. Labit, J. Lamblin, A. Letourneau *et al.*, *Phys. Rev. Lett.* **128**, 061801 (2022).
- [36] I. Goldman, R. N. Mohapatra, and S. Nussinov, *Phys. Rev. D* **100**, 123021 (2019).
- [37] D. McKeen, M. Pospelov, and N. Raj, *Phys. Rev. Lett.* **127**, 061805 (2021).
- [38] I. Goldman, R. N. Mohapatra, S. Nussinov, and Y. Zhang, *Phys. Rev. Lett.* **129**, 061103 (2022).
- [39] Z. Berezhiani, R. Biondi, M. Mannarelli, and F. Tonelli, *Eur. Phys. J. C* **81**, 1036 (2021).
- [40] B. Fornal and B. Grinstein, *Phys. Rev. Lett.* **120**, 191801 (2018); **124**, 219901(E) (2020).
- [41] Z. Berezhiani, *Lett. High Energy Phys.* **2**, 118 (2019).
- [42] A. Bergeron, B. Dionne, and Y. Calzavara, Neutronics Conversion Analyses of the Laue-Langevin Institute (ILL) High Flux Reactor (RHF), Technical Report ANL/GTRI/TM-14/15 (2014), <https://publications.anl.gov/anlpubs/2015/11/122482.pdf>.
- [43] Y. N. Pokotilovski, *Phys. Lett. B* **639**, 214 (2006).
- [44] A. Serebrov, E. B. Aleksandrov, N. Dovator, S. Dmitriev, A. K. Fomin, P. Geltenbort, A. Kharitonov, I. Krasnoschekova, M. Lasakov, A. Murashkin *et al.*, *Nucl. Instrum. Methods Phys. Res., Sect. A* **611**, 137 (2009).
- [45] Z. G. Berezhiani and F. Nesti, *Eur. Phys. J. C* **72**, 1974 (2012).
- [46] Z. Berezhiani, R. Biondi, P. Geltenbort, I. A. Krasnoschekova, V. E. Varlamov, A. V. Vassiljev, and O. M. Zhrebetsov, *Eur. Phys. J. C* **78**, 717 (2018).
- [47] C. Abel *et al.* (nEDM Collaboration), *Phys. Lett. B* **812**, 135993 (2021).
- [48] I. Altarev, C. A. Baker, G. Ban, K. Bodek, M. Daum, P. Fierlinger, P. Geltenbort, K. Green, M. G. Van Der Grinten, E. Gutsmedl *et al.*, *Phys. Rev. D* **80**, 032003 (2009).
- [49] N. J. Ayres, Z. G. Berezhiani, R. Biondi, G. Bison, K. Bodek, V. Bondar, P.-J. Chiu, M. Daum, R. T. Dinani, C. B. Doorenbos *et al.* (nEDM Collaboration), *Symmetry* **14**, 503 (2022).
- [50] P. Mohanmurthy, A. R. Young, J. A. Winger, and G. Zsigmond, *Symmetry* **14**, 487 (2022).
- [51] W. Saenz-Arevalo, G. Ban, J. Chen, P. J. Chiu, B. Clément, M. Guigue, T. Jenke, P. Larue, T. Lefort, O. Naviliat-Cuncic *et al.*, in 56th Rencontres de Moriond on Electroweak Interactions and Unified Theories (2022), pp. 1–4, [arXiv:2206.08721](#).
- [52] G. Ban *et al.*, *Phys. Rev. Lett.* **131**, 191801 (2023).
- [53] Z. G. Berezhiani, M. Frost, Y. Kamyshev, B. Rybolt, and L. Varriano, *Phys. Rev. D* **96**, 035039 (2017).
- [54] U. Schmidt, An experimental limit on neutron mirror-neutron oscillation (2007), <http://inpa-old.lbl.gov/blnv2/files/Saturday/Session13/Schmidt.pdf>.
- [55] J. Redondo and A. Ringwald, *Contemp. Phys.* **52**, 211 (2011).
- [56] C. Stasser, G. Terwagne, J. Lamblin, O. Méplan, G. Pignol, B. Coupé, S. Kalcheva, S. Van Dyck, and M. Sarrazin, *Eur. Phys. J. C* **81**, 17 (2021).
- [57] M. Hostert, D. McKeen, M. Pospelov, and N. Raj, *Phys. Rev. D* **107**, 075034 (2023).
- [58] B. Fornal and B. Grinstein, *Mod. Phys. Lett. A* **35**, 2030019 (2020).

- [59] L. J. Broussard, K. M. Bailey, W. B. Bailey, J. Barrow, B. Chance, C. B. Crawford, L. Crow, L. Debeer-Schmitt, N. Fomin, M. Frost *et al.*, in *Proceedings of the 2017 Meeting of the APS Division of Particles and Fields, DPF 2017* (2017), [arXiv:1710.00767](#).
- [60] L. Broussard, K. Bailey, W. Bailey, J. Barrow, K. Berry, A. Blose, C. Crawford, L. Debeer-Schmitt, M. Frost, A. Galindo-Uribarri *et al.*, *EPJ Web Conf.* **219**, 07002 (2019).
- [61] L. J. Broussard, J. L. Barrow, L. DeBeer-Schmitt, T. Dennis, M. R. Fitzsimmons, M. J. Frost, C. E. Gilbert, F. M. Gonzalez, L. Heilbronn, E. B. Iverson *et al.*, *Phys. Rev. Lett.* **128**, 212503 (2022).
- [62] S. Henderson *et al.*, *Nucl. Instrum. Methods Phys. Res., Sect. A* **763**, 610 (2014).
- [63] J. Haines, T. McManamy, T. Gabriel, R. Battle, K. Chipley, J. Crabtree, L. Jacobs, D. Lousteau, M. Rennich, and B. Riemer, *Nucl. Instrum. Methods Phys. Res., Sect. A* **764**, 94 (2014).
- [64] W. Blokland, G. Armstrong, C. Deibele, J. Pogge, and V. Gaidash, *AIP Conf. Proc.* **868**, 238 (2006).
- [65] H. Ambaye, R. Goyette, A. Parizzi, and F. Klose, *Neutron News* **19**, 11 (2008).
- [66] V. Lauter, H. Ambaye, R. Goyette, W.-T. Hal Lee, and A. Parizzi, *Physica (Amsterdam)* **404B**, 2543 (2009).
- [67] W. Lu, P. Ferguson, E. Iverson, F. Gallmeier, and I. Popova, *J. Nucl. Mater.* **377**, 268 (2008).
- [68] T. Hügle, E. B. Iverson, and F. X. Gallmeier, *J. Phys. Conf. Ser.* **1021**, 012070 (2018).
- [69] I. Levin, *NIST Inorganic Crystal Structure Database (ICSD) 43423* (National Institute of Standards and Technology, 2018), [10.18434/M32147](#).
- [70] B. Yu, J. A. Harder, J. A. Mead, V. Radeka, N. A. Schaknowski, and G. C. Smith, *Nucl. Instrum. Methods Phys. Res., Sect. A* **513** (2003).
- [71] V. Radeka, N. Schaknowski, G. Smith, and B. Yu, *Nucl. Instrum. Methods Phys. Res., Sect. A* **419**, 642 (1998).
- [72] S. Stort, 5 Tesla split-pair superconducting magnet system (2017), <https://cryomagnetics.com/5t-split-pair-superconducting-magnet-system/>.
- [73] T. Sato *et al.*, *J. Nucl. Sci. Technol.* **55**, 684 (2018).
- [74] D. A. Brown *et al.*, *Nucl. Data Sheets* **148**, 1 (2018).
- [75] JENDL-5 ACE library (neutron induced), <https://rpg.jaea.go.jp/main/en/ACE-J50/> (Release Date: 2022/12/9).
- [76] NumPy reference: `numpy.random.multivariate_normal`, <https://numpy.org/doc/stable/reference/random/generated/numpy.random.multivariate-normal.html>.
- [77] R. L. Workman, V. D. Burkert, V. Crede, E. Klempt, U. Thoma, L. Tiator, K. Agashe, G. Aielli, B. C. Allanach, C. Amsler *et al.* (Particle Data Group), *Prog. Theor. Exp. Phys.* **2022**, 083C01 (2022).
- [78] C. Do, W. T. Heller, C. Stanley, F. X. Gallmeier, M. Doucet, and G. S. Smith, *Nucl. Instrum. Methods Phys. Res., Sect. A* **737**, 42 (2014).
- [79] G. J. Feldman and R. D. Cousins, *Phys. Rev. D* **57**, 3873 (1998).
- [80] NIST Neutron scattering length and cross sections (2023), <https://www.ncnr.nist.gov/resources/n-lengths/>.
- [81] H. Abele *et al.*, *Phys. Rep.* **1023**, 1 (2023).
- [82] V. Santoro *et al.*, [arXiv:2311.08326](#).
- [83] L. Broussard, M. Fitzsimmons, M. Frost, Y. Kamyshev, S. Vavra, and H. Ambaye (2020), [10.14461/oncat.data/2462685](#)